Cubes and cuboid Concept

It enhances spatial awareness, pattern recognition, and logical deduction, improving performance in reasoning-based questions. Commonly asked in TCS, Infosys, Wipro, Cognizant, Accenture, Capgemini, Amazon, Adobe, Microsoft, IBM, Google, HCL, Zensar, L&T Infotech, Tech Mahindra, etc.

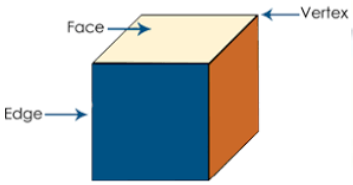
Cubes and cuboid have problems based on surface area and volume and coloring and cutting of cubes, but in reasoning we are going to focus only on coloring and cutting of cube.

Types of problems

1. Finding the number of identical cubes when number of cuts is given.
2. Finding the number of cuts when the number of identical cubes are given.
3. Coloring and cutting cubes.

Key points to remember

a) In a cube there are 6 faces, 12 edges and 8 vertex.



b) A cube can be cut from three dimensions we call first cut 'vertical 1, second cut 'vertical 2' and third one is to be cut horizontally, we call it 'horizontal cut. (Refer diagram below)

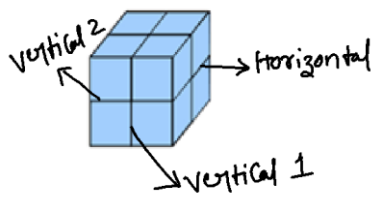
c) In the given diagram it is clearly visible if we put 1 cut on any particular surface, the cube can be divided into two parts.

d) To find the number of identical cubes we can use a trick

e)

	V1		V2		H	
Cuts	1		1		1	
Identical pieces	2	x	2	x	2	= 8 identical cubes

It also shows if the number of cuts on three dimensions are P, Q and R. Then the number of identical pieces will be (P+1), (Q+1) and (R+1). We have to multiply (P+1), (Q+1) and (R+1) to obtain the total number of cubes.



1. Finding the number of identical cubes when the number of cuts is given.

Question1: Find the minimum and maximum number identical pieces using 12 cuts on any surface?

Solution: To Find the minimum number of identical pieces use all the 12 cuts on a single surface like bread cutting, thus the number of identical pieces will be equal to 13.

To find the maximum number of identical pieces try to maximize the number of cuts on all the three dimensions.

So, using four cuts each on V1, V2 and H, the number of identical cubes obtained will be 5 x 5 x 5 = 125.

Trick:

	V1		V2		H	
Cuts	4		4		4	
Pieces	5	x	5	x	5	= 125

Question2: Find the minimum and maximum number identical pieces using 13 cuts on any surface?

Solution: To Find the minimum number of identical pieces use all the 13 cuts on a single surface like bread cutting, thus the number of identical pieces will be equal to 14.

To find the maximum number of identical pieces try to maximize the number of cuts on all the three dimensions. Try to keep as much less difference among V1, V2 and H as possible.

So, using four cuts on two surfaces each as V1, V2 and 5 cuts as H, the number of identical pieces obtained will be 5 x 5 x 6 = 150.

	V1		V2		H	
Cuts	4		4		5	
Pieces	5	x	5	x	6	= 150

Question2: Find the minimum and maximum number identical pieces using 14 cuts on any surface?

Solution: To Find the minimum number of identical pieces use all the 14 cuts on a single surface like bread cutting, thus the number of identical pieces will be equal to 15.

To find the maximum number of identical pieces try to maximize the number of cuts on all the three dimensions. Try to keep as much less difference among V1, V2 and H as possible.

So, using five cuts on two surfaces each as V1, V2 and 4 cuts as H, the number of identical pieces obtained will be 6 x 6 x 5 = 180.

V1	V2		H		
Cuts	5		5		6
Pieces	6	x	6	x	5 = 180

2. Find the number of cuts when the number of identical cubes is given

Question1: Find the maximum and minimum number of cuts required to obtain 64 identical pieces?

Solution: To Find the maximum number cuts, all the cuts can be put on single surface like bread cutting, in that case if total identical pieces are 64 number cuts required should be 63.

To find the minimum number of cuts factorize 64 in form of V1, V2 and H with least gap among them

	V1		V2		H
Pieces	4	x	4	x	4
Cuts	(4-1) +		(4-1) +		(4-1) = 9 cuts

Question: Find the maximum and minimum number of cuts required to obtain 120 identical pieces?

Solution: To Find the maximum number cuts, all the cuts can be put on single surface like bread cutting, in that case if total identical pieces are 120, then the number cuts required should be 119.

To find the minimum number of cuts factorize 120 in form of V1, V2 and H with least gap among them,

So there can be more possible factors pair like : 4 x 5 x 6, 10 x 6 x 2 and 8 x 5 x 3 etc. but 4 x 5 x 6 is with least difference, will pick that only.

	V1		V2		H
Pieces	4	x	5	x	6
Cuts	(4-1) +		(5-1) +		(6-1) = 12 cuts

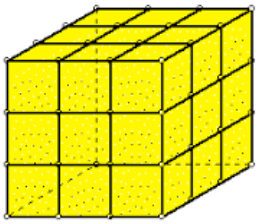
3. Coloring and cutting of cubes

Some important tricks to remember for coloring and cutting cube

- For a cube of side n*n*n painted on all sides which is uniformly cut into smaller cubes of dimension 1 x 1 x 1,
- Number of cubes with 0 side painted = $(n - 2)^3$
- Number of cubes with 1 side painted = $6 (n - 2)^2$
- Number of cubes with 2 sides painted= $12 (n - 2)$
- Number of cubes with 3 sides painted= 8 (always)
- For a cuboid of dimension a*b*c painted on all sides which is cut into smaller cubes of dimension 1*1*1,
- Number of cubes with 0 side painted= $(a - 2) (b - 2) (c - 2)$
- Number of cubes with 1 sides painted $=2 [(a - 2)(b - 2) + (b - 2)(c - 2) + (a - 2)(c - 2)]$
- Number of cubes with 2 sides painted= $4(a + b + c - 6)$
- Number of cubes with 3 sides painted= 8

Direction for question (1-7): A cube having a side of 12 cm is painted yellow on all the faces and then cut into smaller cubes of 4 cm each. Find the total number of smaller cubes so obtained. Answer the following questions:

1) Find the total number of smaller cubes obtained?
2) Find the number of cubes with 3 faces painted?
3) Find the number of cubes with 2 faces painted?
4) Find the number of cubes with 1 face painted?
5) Find the number of cubes 0 face painted?
6) Find the number of cubes with at least 2 faces painted?
7) Find the number of cubes with at most 1 face painted?



Solution: 1. The number of cubes obtained 3 x 3 x 3 = 27

2. The number of cubes with 3 faces painted are at the vertex which 8.

3. To find the number cubes with 2 faces painted use the following formula 12 (n - 2) = 12 (3-2) = 12

4. To find the number cubes with 1 face painted use the following formula 6 $(n - 2)^2$

$6 (3 - 2)^2 = 6$

5. To find the number cubes with 0 face painted use the following formula = $(n - 2)^3$

$(3 - 2)^3 = 1$

6. To find the number of cubes with at least 2 faces painted, we can add cubes with three and 2 faces painted = 8 + 12 = 20

7. To find the number of cubes with at most 1 face painted, we can add cubes with 0 and 1 face painted = 1+6 = 7.