

Average concepts

The average is defined as the mean value which is equal to the ratio of the sum of the number of a given set of values to the total number of values present in the set. The average is basically the mean of the values which are represented by \bar{x} . It is also denoted by the symbol ' μ '.

The main purpose of average is to make all the observations same.

$$\text{Average formula} = \frac{\text{Sum of observation}}{\text{Number of observation}}$$

Question: Find the average of the following numbers 24, 29, 36, 47, 60

Solution: $\frac{24+29+36+47+60}{5} = 39.2$

Types of average/mean

Arithmetic mean: The arithmetic mean is the sum of the number and divided by n, where n is the total numbers,

Geometric mean: In geometric mean, we find the product of all the observations and then find the nth root of the product, provided that n is number of observations.

Harmonic mean: The harmonic mean is defined as the reciprocal of the average of the reciprocals of the given data values.

Average of Arithmetic progression (AP) series

AP is the series where the common difference among the observation is always the same. Following are the points to remember:

1) Average can be done by three methods:

- i) Average of AP = $\frac{A+L}{2}$
- ii) Middle term of AP is the average
- iii) Sum of AP = $\frac{A+L}{2} \times n$

Question: Find the average and sum of the following series: 10, 15, 20, 25, 30

Solution: By first method = $A = 10, L = 30 \quad \frac{10+30}{2} = 20$

By second method 20 is the average

Using third method, sum formula $\frac{A+L}{2} \times n = \frac{10+30}{2} \times 5 = 20 \times 5 = 100$

Note: In case, if n is even then mid-point of AP is the average

Ex – 10, 20, 30, 40 the mid-point is 25 which is mean of the given series.

Problems in average

There can be four types of questions asked in averages.

- 1) Change in average due to joining of new numbers.
- 2) Change in average due to leaving of numbers.
- 3) Change in average due to replacement.
- 4) Group average.

Change in average due to join: When a new number joins, there can be increase or decrease in average depending upon the number which is joining.

Question 1: The mean of 5 numbers is 29, if one new number joins with value 47 what will be the new average.

Solution: If mean of 5 number is 29, then initial sum should be $29 \times 5 = 145$

If one new number joins which is 47, the new sum becomes $= 145 + 47 = 192$

Since the numbers (n) are 6 now, new average $= 192 / 6 = 32$

Question 2: The mean of 5 numbers is 44, due to joining of a new number the new average become 45. What is the new number joined?

Solution: If mean of 5 number is 44, then initial sum should be $44 \times 5 = 220$

After joining sum should be $= 45 \times 6 = 270$

The number joined $= 270 - 220 = 50$

Change in average due to leave- When a number leaves, there can be increase or decrease in average depending upon the number which is leaving.

Question 1: The mean of 5 numbers is 29, if one number leaves with value 33 what will be the new average.

Solution: If mean of 5 number is 29, then initial sum should be $29 \times 5 = 145$

When a number leaves which equal to 33, new sum $= 145 - 33 = 112$

Since number (n) = 4 now

So, new mean $= 112 / 4 = 28$

Question2: The mean of 6 observation is 60, when a number is removed the average becomes 62. What is the number which is being removed?

Solution: If mean of 6 number is 60, then initial sum should be $60 \times 6 = 360$

After removing one number new sum = $62 \times 5 = 310$

So, the number removed = $360 - 310 = 50$.

Change in average due to replacement: When a number is replaced by another number, there can be increase or decrease in average depending upon the positive or negative difference between the numbers which are replacing each other.

Question 1: The mean of 5 numbers is 29, if one number 15 is replaced by another number 55. What should be the new average?

Solution: If mean of 5 number is 29, then initial sum should be $29 \times 5 = 145$

When 15 leaves = $145 - 15 = 130$

And replaced by 55 = $130 + 55 = 185$

So, new mean = $185 / 5 = 37$.

Question 2: The mean of 5 numbers is 35, if one number 15 is replaced by another number x , hence the average of the series become 38. What should be the value of x ?

Solution: If mean of 5 number is 35, then initial sum should be $35 \times 5 = 175$

When 15 leaves, the sum should be = $175 - 15 = 160$

But the new sum should be = $38 \times 5 = 190$.

So the number $x = 190 - 160 = 30$.

Group average-In group average, you are given average of two or more different groups, and you are being asked to find the combined average of the whole group.

Question: In a class, there are 45 girls and 55 boys the average score of girls is 40 marks and that of the boys is 50. Find the average score of the whole class?

Solution: The sum of score of girls = $45 \times 40 = 1800$

The sum of scores of the boys = $55 \times 50 = 2750$

Sum of the whole class = $1800 + 2750 = 4550$

Average of the whole class = $4550 / 100 = 45.5$